PHYSICS

DPP
DAILY PRACTICE PROBLEMS

DPP No. 68

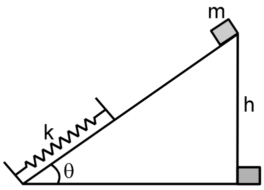
Total Marks : 22

Max. Time : 24 min.

Topics : Work, Power and Energy, Rigid Body Dynamics, Center of Mass

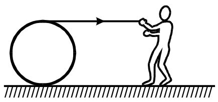
Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.3	(3 marks, 3 min.)	[6, 6]
Multiple choice objective ('–1' negative marking) Q.4 to Q.5	(4 marks, 4 min.)	[8, 8]
Subjective Questions ('–1' negative marking) Q.6 to Q.7	(4 marks, 5 min.)	[8, 10]

1. A body of mass m released from a height h on a smooth inclined plane that is shown in the figure. The following can be true about the velocity of the block knowing that the wedge is fixed.



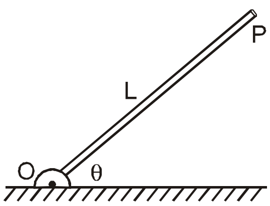
(A) v is highest when it just touches the spring
(B) v is highest when it compresses the spring by some amount
(C) v is highest when the spring comes back to natural position
(D) none of these

2. A man pulls a solid cylinder (initially at rest) horizontally by a massless string. The string is wrapped on the cylinder and the cylinder performs pure rolling. Mass of the cylinder is 100 kg, radius is π metre & tension in string is 100 N. Then the angular speed of the cylinder after one revolution will be :



(A) 4 rad /sec
(B) $\frac{4}{\sqrt{3}}$ rad/ sec
(C) $\frac{4}{3}$ rad/ sec
(D) none of these

3. A uniform pole of length L and mass M is pivoted on the ground with a frictionless hinge O. The pole is free to rotate without friction about an horizontal axis passing through O and normal to plane of the page.The pole makes an angle θ with the horizontal. The pole is released from rest in the position shown, then linear acceleration of the free end (P) of the pole just after its release would be :



(A) $\frac{2}{3}$ g cosθ
(B) $\frac{2}{3}$ g
(C) g
(D) $\frac{3}{2}$ g cosθ

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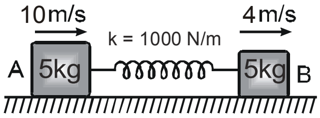
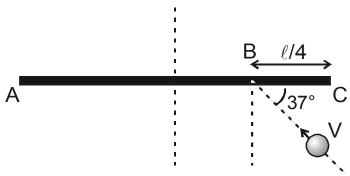
4. Two blocks A (5kg) and B(5kg) attached to the ends of a spring constant 1000 N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 10m/s and 4 m/s along the line of the spring in the same direction are imparted to A and B then



(A) when the extension of the spring is maximum the velocities of A and B are same.
(B) the maximum extension of the spring is 30cm.
(C) the first maximum compression occurs $\pi/56$ seconds after start.
(D) maximum compression and maximum extension occur alternately.

5. A rod AC of length ℓ and mass m is kept on a horizontal smooth plane. It is free to rotate and move. A particle of same mass m moving on the plane with velocity v strikes rod at point B making angle 37^0 with the rod. The collision is elastic. After collision :



(A) The angular velocity of the rod will be $\frac{72}{55}\frac{v}{\ell}$

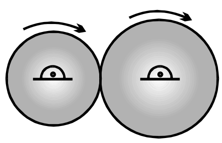
(B) The centre of the rod will travel a distance $\frac{\pi \ell}{3}$ in the time in which it makes half rotation

(C) Impulse of the impact force is $\frac{24mV}{55}$

(D) None of these

6. A block of dimensions $a \times a \times 2a$ is kept on an inclined plane of inclination 37°. The longer side is perpendicular to the plane. The co-efficient of friction between the block and the plane is 0.8. By numerical analysis find whether the block will topple or not.

7. Two separate cylinders of masses m (= 1 kg) & 4 m & radii R (= 10 cm) and 2 R rotating in clockwise direction with ω_1 = 100 rad/sec. and ω_2 = 200 rad/sec respectively. Now they are held in contact with each other as in figure. Determine their angular velocities after the slipping between the cylinders stops.



Answers Key

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1. (B) **2.** (B) **3.** (D) **4.** (A), (B), (D)
5. (A), (B), (C)
6. Since torque is not balanced, it will topple.
7. 300rad/sec., 150 rad/sec

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Hint & Solutions

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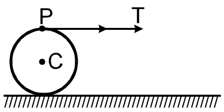
1. Velocity is maximum when acceleration is zero. It means net force is zero. Net force is zero after some compression.

2. The cylinder rolls without slipping, hence no work is being done by friction. In one complete revolution the centre C of the cylinder moves by $2\pi R$ (R is radius of cylinder) and the top most point P of the cylinder moves by $4\pi R$.



$v_{cm} = R\omega$ (from constraint)

Applying work energy theorem

Work done by T= increase in kinetic energy of cylinder

$$T \times 4\pi R = \frac{1}{2} I_{cm}\,\omega^2 + \frac{1}{2} m v_{cm}^{\ 2} = \frac{1}{2}\left(\frac{1}{2} mR^2\right)\omega^2$$

$$+ \frac{1}{2} mR^2\ \omega^2$$

solving we get $\omega = \frac{4}{\sqrt{3}}$ rad/ sec

3. About point O

Torque $\tau = I\alpha$

$$Mg\left(\frac{L}{2}\cos\theta\right) = \frac{ML^2}{3}\alpha \quad \Rightarrow \quad \frac{3}{2}\frac{g}{L}\cos\theta = \alpha$$

Initially centripetal acceleration of point P is zero

$$(\because\ a_c = \frac{v^2}{r} = \frac{0}{r} = 0\ )$$

Acceleration of point P is $\sqrt{a_C^2 + a_t^2}$

$$= a_t = L\alpha = \frac{3}{2} g\cos\theta$$

4. $\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(V_1 - V_2)^2 = \frac{1}{2} kx^2$

$$\frac{1}{2}\frac{(5)(5)}{5+5}(10-4)^2 = \frac{1}{2} \times 1000\ x^2$$

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$$\frac{(25)(36)\times 10^{-1}}{1000} = x^2$$

$$\frac{(25)(36)}{10000} = x^2$$

$$\frac{(5)(6)}{10} = x^2$$

x = 0.30 m

Also $\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{1000}{\frac{(5)(5)}{5+5}}}$

ω = 20 sec.

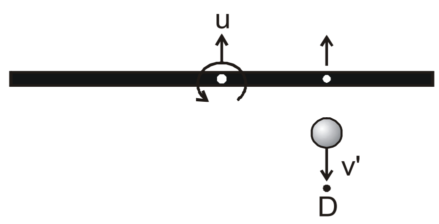
$$T = \frac{2\pi}{20} = \frac{\pi}{10}$$

The first maximum compression occurs $\frac{T}{4} = \frac{\pi}{40}$ sec. after start.

5. The ball has V', component of its velocity perpendicular to the length of rod immediately after the collision. u is velocity of COM of the rod and ω is angular velocity of the rod, just after collision. The ball strikes the rod with speed vcos53° in perpendicular direction and its component along the length of the rod after the collision is unchanged.

Using for the point of collision.

Velocity of separation = Velocity of approach



$$\Rightarrow \frac{3V}{5} = \left(\frac{\omega\ell}{4} + u\right) + V' \quad .... (1)$$

Conserving linear momentum (of rod + particle), in the direction $\perp$ to the rod.

$$mV.\frac{3}{5} = mu - mV' \quad ....(2)$$

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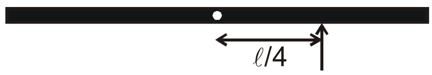
Conserving angular moment about point 'D' as shown in the figure

$$0 = 0 + \left[mu\frac{\ell}{4} - \frac{m\ell^2}{12}\omega\right] \Rightarrow u = \frac{\omega\ell}{3} \quad ....(3)$$

By solving

$$u = \frac{24V}{55}, \; w = \frac{72V}{55\ell}$$

Time taken to rotate by π angle t = $\frac{}{\omega}$



In the same time, distance travelled $= u_2.t \;\; = \frac{\ell}{}$

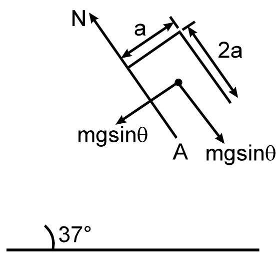
Using angulr impulse-angular momentum equation.

$$\int N.dt.\frac{\ell}{4} = \frac{m\ell^2}{4}.\frac{72V}{55\ell} \Rightarrow \int N.dt = \frac{24mV}{55}$$

or $\begin{cases} \text{using impulse} - \text{momentum equation on Rod} \\ \int Ndt = mu = \frac{24mv}{55} \end{cases}$

6. If ever it will topple, it will topple about A. It can be verified that the block is not sliding.

Now, τ_A = mg sin θ × a – mg cos θ × $\frac{a}{}$



$= \tau_A \qquad = \frac{mag}{}$ which is non–zero.

Since torque is not balanced, it will topple.

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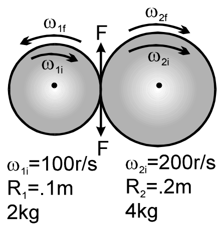
7. Final direction of motions are shown by ω_{1f} & ω_{2f}



Now,

$$\alpha_1 = \frac{\omega_{f1} + \varepsilon_{i1}}{t} \qquad \alpha_2 = \frac{\omega_{i2} + \varepsilon_{f2}}{t}$$

and $FR_1 + I_1\alpha_2$ (torque equation. of friction)

$FR_2 = I_2\alpha_2$

Dividing $\frac{R_1}{R_2} = \frac{\ell_1\,\alpha_1}{\ell_2\,\alpha_2}$

$$\Rightarrow \quad \frac{I_1}{I_2} \cdot \frac{\omega_{f_1} + \omega_{i_1}}{\omega_{i_2} - \omega_{f_2}} = \frac{R_1}{R_2}$$

For pt. of contact when slipping stops

$R_1\,\omega_{f_1} = R_2\,\omega_{f_2}$

$$\frac{\mu_1 R_1{}^2\,\ell_2}{\mu_2 R_2{}^2\,\ell_2} \cdot \frac{\omega_{f_1} + \omega_{i_1}}{\omega_{i_2} - \frac{R}{R_2}} = \frac{R_1}{R_2}$$

$$\Rightarrow \quad \omega_{f_1} = \frac{\mu_2 R_2 \omega_{i_2} - \mu_1 R_1 \omega_{i_1}}{\mu_2 R_1 + \mu_1 R_1}$$

$$= \frac{4 \times .2 \times 200 - 1 \times .1 \times 100}{.4 \times +.1} = 300 \text{ r/s}$$

$$\omega_{f_2} = \frac{R_1 \omega_{f_1}}{R_2} = \frac{R}{2R} \times 300 \quad = 150 \text{ rad/sec.}$$

[Ans.: 300rad/sec., 150 rad/sec.]

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